MATH 526: TOPICS IN TOPOLOGY, Spring 2018
MW 1:20 PM - 2:40 PM, Exley 638, Instructor: Alyson Hildum

Syllabus.

Math 526 will be a topics course in topology focusing on low-dimensional manifolds, specifically in dimension 4. The prerequisites are Math 523 and 524 or the equivalent. I plan to teach from the book *4–Manifolds and Kirby Calculus*, by Gompf and Stipsicz, supplemented by other materials listed below. The goal of this course will be to gain a basic understanding of properties and constructions of manifolds in low (below 5) dimensions, focusing on the wild behavior of manifolds in dimension 4. We will start with some preliminaries, including a basic introduction to cohomology, all of which will depend on the combined background of the class. I'd like to cover chapters 1, and 4 through 6 of Gompf and Stipsicz, and with any leftover time we will cover additional topics depending on the interest of the class. I, in particular, am interested in learning more about exotic manifolds, so chapter 9 is a further possibility.

Grading will be based on participation, occasional homework assignments, and class presentations.

A rough partial outline of the Gompf-Stipsicz book is provided below:

(1) Introduction: manifolds, 4–manifolds, examples

(2) Surfaces in 4–manifolds: surfaces in $\mathbb{C}P^2$, blow-ups, desingularization of curves, intro to gauge theory

(3) Complex surfaces: E(1) and fiber sum, other constructions of elliptic fibrations, log transformations, classification of complex surfaces

(4) Handlebodies and Kirby diagrams: handles, handle decompositions, Heegaard splittings, Kirby diagrams, Linking numbers and framings, examples

(5) Kirby Calculus: Handle moves, surgery, Dehn surgery, 1-handles revisited, relative Kirby Calculus, spin structures

(6) More examples: plumbings, embedded surfaces, branched covers

(7) Branched covers and resolutions

(8) Elliptic and Lefshetz fibrations

(9) Cobordisms, h-cobordisms, and exotic $\mathbb{R}^4$'s

(10) Symplectic 4–manifolds

(11) Stein surfaces

Additional sources. I own or have checked out copies of all the following texts, including Gompf and Stipsicz. If you do not wish to purchase any of the following textbooks, I can loan the books out to you short-term.

1. Hatcher, *Algebraic Topology*

2. Scorpan, *The Wild World of 4–Manifolds*

3. Milnor and Stasheff, *Characteristic Classes*

4. Guilleman and Pollack, *Differential Topology*

5. Kirby, *The Topology of 4–Manifolds*

6. Freedman and Quinn, *Topology of 4–Manifolds*